Angles

Penalty shoot out

World Cup football games end with a penalty shootout if the teams are still level on goals at the end. Each team has to send out players in turn to kick a ball from a fixed spot into the goal.

If you were the player taking a critical penalty kick that would either give your team victory in the World Cup final or send them home in despair, you'd want to have the best chance of getting your kick past the opposing goalkeeper and into the goal!

It's very hard for a goalkeeper to reach the top left and right corners of the goal. But it's also very difficult to kick the ball accurately into those corners. One of the reasons for this is that a small difference in the **angle** that you kick the ball at makes a huge difference in the height of the ball above the ground when it reaches the goal.

A few degrees wrong in the angle one way and you'll send the ball way above the bar and into the crowd. A few degrees wrong in the angle the other way and the ball will pass low enough for the goalkeeper to be able to catch it easily if they dive that way.

In international football, the goalposts are 24 feet apart and the crossbar connecting them is 8 feet above the ground. You take a penalty kick from a spot 36 feet away from the spot halfway between the goalposts.

Let's convert these figures to metres using the conversion factor 1 foot = 0.3048 metres. In metric units, the goalposts are 7.32 metres apart, the crossbar is 2.44 metres above the ground and the penalty spot is 10.97 metres back.

If you're aiming for the corner of the goal you've got to kick a horizontal distance which is equal to the third side of the triangle shown. It can be calculated using Pythagoras' theorem: it's approximately 11.56 metres.

3.66m 10.97m A
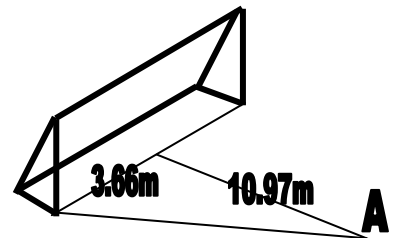
1. Look at the angle at corner A of this triangle. Using any method you prefer, estimate this angle to the nearest degree. The diagram is not to scale!

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2. Suppose that you take the risky shot and aim your kick along the bold line.
    Assuming you can aim your kick accurately, the ball will be heading for the left goalpost.
    Let's assume also that you can kick the ball so that it covers 20 metres per second.

(a) If you kick the ball straight up into the air, how long will it take to reach the goal?

(b) If you kick the ball horizontally along the ground, how long will it take to reach the goal?
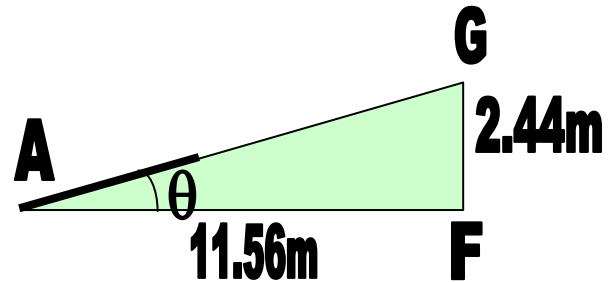
What's the right angle to kick the ball at?

The triangle shows the angle involved in taking a penalty kick into the top left hand corner of the goal.

The right hand side represents the height of the goal, which is 8 feet, or 2.44 metres.

The horizontal side represents the distance to the corner of the goal, which is 11.56 metres.

1. If you stand at A, what is the angle **θ** between the foot of the goalpost at F and the top of the goalpost at G? (There are several ways to answer this. One would be to draw an accurate scale diagram of the triangle and then measure the angle.)

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2. Suppose that you kick the ball directly at G, so that it starts off heading straight towards G. While it is in flight it will be affected by gravity pulling it down towards the ground. Will the ball hit the line FG below or above the point G?

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3. Suppose that you want to kick the ball so that it hits G, the top of the goal post. Given your answer to Question 2, should you kick the ball at angle **θ** or at a smaller or larger angle? Explain your answer.

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Project

You meet up with Nadia, who's studied mechanics at A-level. She tells you that if you kick the ball at angle α and at 20 metres per second then the height **y** of the ball when it reaches the goal can be modelled by the following equation.
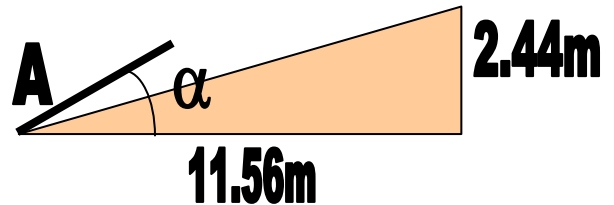
$$y = 11.56 \tan \alpha - \frac{1.637}{(\cos \alpha)^2}$$

If this gives you a negative answer it means that the ball bounced on the ground at least once before it reached the goal. If it gives a positive answer then that's the height above the ground.

Suppose that you kick the ball at the following angles α and use the equation to find out the height of the ball when it reaches the goal. Remember if it's over 2.44m then it's too high.

Angle α (in degrees)	Height **y** (in metres)	Did the ball bounce?	Was it too high?
5 degrees			
10 degrees			
15 degrees			
20 degrees			
25 degrees			
30 degrees			

Choose angles for the final four rows of the table to see if you can find the perfect angle to hit the top of the goalpost to the nearest degree. How does this compare with the angle **θ**?

What other factors would affect the angle you need to kick the ball at?