

Picture Perfect Processing



Mathematical image processing techniques make it possible for us to capture, transmit and store photographs and video. They also let us restore noisy or damaged images and extract useful information from visual data.

Our world is full of cameras and other imaging sensors producing millions of photographs, video clips and digital 3D models every single day. Imaging technology is constantly evolving, with new products and applications launched each year, and the rate of change is so fast that the digital cameras found in modern mobile phones now provide better quality images than the top-of-the-range digital cameras of the late 1990s.

New technology by itself is not enough to provide all of these advances though - it must be supported by advances in the mathematical software operating behind the scenes. Image processing is a young mathematical field, but all modern imaging technology depends on it for many aspects of image capture, transmission and storage. The field's techniques can also be used to clean up noisy or damaged pictures and extract usable information from scientific images.

Image processing began in the 1960s with the development of an algorithm called the "fast Fourier transform" (FFT). The Fourier transform is a 200 year old mathematical operation for splitting a signal into distinct parts that can be processed separately - a

method, opening the doors for digital image processing.

as a signal, but this is a more complicated process due to the large amount of data involved. Using the Fourier transform on images in this way wasn't feasible until the FFT algorithm provided a fast computerised

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Since then, more advanced techniques have been developed. The Fourier transform works well for images with smooth, curved features, but is less suitable for straight edges

and right angles. These features are better handled by alternative approaches such as the wavelet decomposition, which is used to store and transmit the billions of JPEG images on the internet and was developed in the 1980s.

Chris Budd, a mathematician at the University of Bath, working closely with the INVERT Centre (also at Bath), uses image processing techniques such as these for a wide range of applications in many scientific areas. For example, the very edge of the Earth's atmosphere, known as the ionosphere, is made up of constantly shifting electrically charged particles. It is important to monitor the changing conditions in this atmospheric layer, as they can interfere with radio communications, damage satellites and overload electrical grids, and image processing techniques can further our understanding.

The University of Bath's work in this area uses signals from GPS satellites to build up an image of the ionosphere by measuring how the signals are delayed as they pass through the atmosphere; the longer the delay the greater the electrical activity in that area. Since we know when the GPS signals should arrive it is possible to clean up the signal using image processing techniques.

Comparing the clean signal to the delayed



signal gives a measure of the electrical activity in the atmosphere between the GPS satellite and GPS receiver. By using multiple satellite signals and lots of Earth-based receivers, combined with a mathematical model of the atmosphere, it's possible to build up a useful map of the ionosphere that can indicate when and where problems might occur.

The Bath team, working in collaboration with various other groups around the world, has also helped to develop a new technique, known as diagnostic radioentomology, which produces 3D images of the inside of a bee hive in real-time without the need to open the hive or otherwise disrupt the bees. It uses X-rays to visualise the inside of the hive, much like we use X-rays in a hospital, but because bees are much smaller than humans imaging the hive with traditional X-ray techniques requires much higher doses of radiation. These high doses can be harmful to the bees, however, by refining and speeding up the techniques for solving the equations involved, it is possible to find solutions faster, making possible it to extract more information from lower levels of radiation exposure, thus making it safer for the bees.

This new technique could help explain why bees are currently dying out, a phenomenon that is yet to be fully explained by scientists. The health of bees and their hives is of great concern to humans, because many plants, including agricultural crops, depend on the



insects for pollination. The radioentomology technique is powerful enough to track the movements of individual bees within the hive, allowing biologists to learn much more about their behaviour in an effort to solve the problem.

In both these and many other applications of image processing, mathematicians, physicists and electrical engineers are working with experts in outside fields to help them extract useful information in difficult circumstances. These image processing techniques must be tailored to each situation, but in the future

it is very possible that a computer could be taught to understand the general principles of any image. Humans can easily look at an image of a field and pick out features such as individual trees, but this is currently a very difficult task to automate. Just as the fast Fourier transform gave birth to the field of image processing, future techniques could make this advanced task a reality and lead to an exciting new range of applications.

TECHNICAL SUPPLEMENT

Imaging the ionosphere

The ionosphere is the outer shell of the Earth's atmosphere and is thus a near-vacuum, but it is not entirely empty. The relatively small number of atoms that do exist within the ionosphere are under constant bombardment from solar radiation and high energy particles, causing them to lose electrons and become positively charged ions.

Measuring these constantly changing conditions is difficult, but the team of Chris Budd and the INVERT Centre at the University of Bath have devised a low-cost method known as MIDAS (Multi-Instrument Data Analysis System). It analyses the delay of the known signals from GPS satellites and calculates the electron density in a line directly between the satellite and the GPS receiver.

The system divides the atmosphere into a grid of small geographical regions. Estimating the electron density from the full set of signal delays involves solving a large set of linked equations, a task for which mathematical inversion methods are well suited. The densities are then mapped back on to the geographical grid to produce a useful image. The resulting electron density can be compared to a mathematical model of the physics of the ionosphere, to confirm whether the values are physically realistic.

References

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