

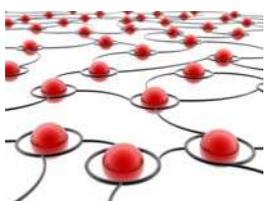
# Simultaneous Equations



Starter worksheet

Simultaneous equations are a branch of algebra where you may be asked to solve problems such as the following.

**Anna goes to the shop and buys three apples and two pears, which costs her £1.46.  
Bilal goes to the same shop and buys one apple and four pears, which costs him £1.52.  
How much does an apple cost?**



The first question you may find yourself asking is – why don't you go to the shop yourself and find out? It's not usual in real life to have to solve this kind of problem. But this kind of problem – on a much more complicated scale! – is solved by companies running web search engines all the time. When you type in “the world's tallest man” to your favourite search engine, how does it work out which sites to show you? We'll look at how that works on the next sheet.

The second question you should ask is how you can solve this problem of apples and pears, so that you can understand the maths underlying internet searches.

There are lots of ways to solve the problem. Here are a few ways.

## Method 1. (Reasoning in words)

Suppose that Bilal had bought three times the quantity of fruit that he bought – three apples and twelve pears. Then he'd spend three times £1.52, which is £4.56. Now Anna's three apples and two pears cost £1.46, so the difference between these will be the cost of ten pears. Since  $£4.56 - £1.46 = £3.10$ , a single pear costs 31p. Therefore four pears cost £1.24, so the apple Bilal bought must cost  $£1.52 - £1.24$ , which is 28p.

## Method 2. (The same reasoning, but with algebraic symbols)

All of the above working can be put into an algebraic form.

Let  $a$  be the cost of an apple and  $b$  the cost of a pear, in pence.

Therefore we have two equations:  $3a + 2b = 146$  and  $a + 4b = 152$ .

Multiply the second equation by 3 on both sides to give  $3a + 12b = 456$ .

Subtract the first equation from the second as follows:  $3a + 12b - (3a + 2b) = 456 - 146$ .

Simplify to give  $10b = 310$  and divide by 10 on both sides to give  $b = 31$ .

Then substitute  $b = 31$  into  $a + 4b = 152$  to give the equation  $a = 152 - 4 \times 31 = 28$ .

So an apple costs 28p.

## Method 3. (Estimation)

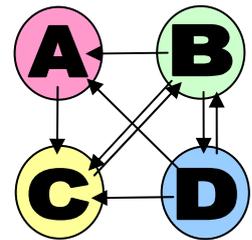
Notice that both Anna and Bilal bought five pieces of fruit, and that the total costs were very similar: £1.46 and £1.52. Therefore each piece of fruit must cost roughly  $£1.50$  divided by 5, or 30p. Try different values for the cost of an apple and the cost of a pear until you see that 28p and 31p works.

1. Find another way of working out the cost of an apple. Which method do you prefer?
2. Nadia goes to the shop and buys three bananas and a mango, which costs her £1.89. Oliver goes to the same shop and buys eight bananas and two mangos, which costs him £4.28.
  - (a) How much does a mango cost? Use any method you like to calculate the answer.
  - (b) Use a different method to calculate the cost of a mango.

# Simultaneous Equations

Simultaneous equations aren't just used to figure out the cost of apples and bananas. If you use an internet search engine to find websites then you're using the results of solving simultaneous equations. However these calculations involve millions of equations rather than just two or three. Although there are powerful computers to do the hard work, they need to be programmed to carry out the right algorithm. Understanding **how** to solve the equations in the fastest way possible makes all the difference for companies running a search engine: being very successful or going out of business.

To understand a bit about how search engines work, we'll look at one simplified example. Suppose that the whole internet consists of just four pages. In the diagram, an arrow means that the webpage at the start of the arrow contains a link to the webpage at the end of the arrow. So the webpage British Beaches has a link to Amazing Adventures but not the other way around.



**Webpage A:** Amazing Adventures

**Webpage B:** British Beaches

**Webpage C:** Caribbean Cruises

**Webpage D:** Dixon's Directory

A good webpage will usually be connected to by lots of links from other webpages. This isn't the only thing that search engines look at when deciding which webpages to suggest, but it's a start.

1. How many links point to each of the above webpages? Which webpage has the most links to it?

Search engines take this idea one step further and give webpages a higher value if they are linked to by webpages which themselves have a higher value. The next two questions give an idea of how this is done.

2. In the grid on the left below, write 1 in row A and column B if there's an arrow pointing from B to A, and write 0 if there isn't an arrow from B to A. Do the same for all the boxes.

The grid on the right is calculated by taking each column of the first grid in turn and dividing each of the entries by the total gained by adding up the numbers in that column. Check this against your answer to Q2.

	A	B	C	D
A	0			
B		0		
C			0	
D				0

	A	B	C	D
A	0	$\frac{1}{3}$	0	$\frac{1}{3}$
B	0	0	1	$\frac{1}{3}$
C	1	$\frac{1}{3}$	0	$\frac{1}{3}$
D	0	$\frac{1}{3}$	0	0

3. This grid will help us work out a value for each webpage. Take four variables **a**, **b**, **c** and **d**. These will have values between 0 and 1, and are the quality values calculated for Webpage A, B, C and D respectively. Take four simultaneous equations so that the coefficients come from the grid.

First row:  $\frac{1}{3}x b + \frac{1}{3}x d = a$

Second row:  $c + \frac{1}{3}x d = b$

Third row:  $a + \frac{1}{3}x b + \frac{1}{3}x d = c$

Fourth row:  $\frac{1}{3}x b = d$

Assuming that  $a + b + c + d = 1$ , solve the equations to find the values of **a**, **b**, **c** and **d**. If the search engine lists the pages in order of their value with largest first, what order will they be listed in?