

Sine waves in music

Keyboard musical note	Frequency
F2	87.31
F2#	92.50
G2	98.00
G2#	103.83
A2	110.00
A2#	116.54
B2	123.47
C3	130.81
C3#	138.59
D3	146.83
D3#	155.56
E3	164.81
F3	174.61
F3#	185.00
G3	196.00
G3#	207.65
A3	220.00
A3#	233.08
B3	246.94
C4	261.63
C4#	277.18
D4	293.66
D4#	311.13
E4	329.63
F4	349.23
F4#	369.99
G4	392.00
G4#	415.30
A4	440.00
A4#	466.16
B4	493.88
C5	523.25
C5#	554.37
D5	587.33
D5#	622.25
E5	659.26
F5	698.46
F5#	739.99
G5	783.99
G5#	830.61
A5	880.00
A5#	932.33
B5	987.77
C6	1046.50

The decimal numbers are the frequencies of those notes in Hertz (the number of peaks of the sound wave in a single second).



The table gives the names and frequencies of various musical notes that you would find on a piano or keyboard. C4 is middle C and is a note that almost all female singers and most male singers can sing comfortably.

The white keys are called A, B, C, D, E, F, G then this pattern repeats again. The lowest note called A in this table is called A2, then the next time we come to a note A it's called A3, and so on. A grand piano keyboard would have keys from A0 to C8 on it.

The black keys are called A#, C#, D#, F# and G#, where the # symbol is called "sharp" and means that that note is slightly higher. So C4# is a slightly higher note than C4.

Look at the keyboard and answer the following questions.

1. If you divide the frequency of A5 by the frequency of A4, what do you get?
2. Which other pairs of frequencies give this result when you divide one by the other?
3. If you divide the frequency of A4# by the frequency of A4, what do you get?
4. Which other pairs of frequencies give this result?
5. What would the frequency of C7 be?
6. What would the frequency of C6# be?
7. Which note has frequency 3520 Hertz?

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Keyboard note	Frequency	Frequency divided by 261.63
C4	261.63	
C4#	277.18	
D4	293.66	
D4#	311.13	
E4	329.63	
F4	349.23	
F4#	369.99	
G4	392.00	
G4#	415.30	
A4	440.00	
A4#	466.16	
B4	493.88	
C5	523.25	

Any pair of notes where one note's frequency is exactly double the other note's frequency are said to be an **octave** apart. The two notes sound good when played together.

The Greek mathematician Pythagoras is famous for his theorem about triangles, but he is also thought to have discovered that notes which sound good together tend to have frequencies whose ratio is a fraction involving two small whole numbers.

Complete the third column of the table and use it to answer the following questions.

1. Try to find two notes so that the frequency of one of them is double the other.
2. Which note has frequency which is close to being $\frac{3}{2}$ of the frequency of C4?
3. Which note has frequency which is close to being $\frac{4}{3}$ of the frequency of C4?
4. Which note has frequency which is close to being $\frac{5}{3}$ of the frequency of C4?
5. Is C4# / C4 close to a fraction involving two small whole numbers?

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Advanced worksheet

Notes on a piano or keyboard are an approximation to the frequencies that would really sound best together. For example, think about G4 (the G above middle C) and suppose that you were trying to work out the best frequency to tune that note to.

- G4 sounds best when played with C4 if its frequency is exactly $3/2$ times 261.63, which would be 392.45.
- But it sounds best when played with D4 if its frequency is exactly $4/3$ times 293.66, which would be 391.55.

You could change the frequency of D4, but then some other pair of notes would sound worse. On a violin or many other instruments, musicians play slightly different frequencies for G4 depending on the other notes that are being played at the time.

1. Calculate the ratio of frequencies between any pair of consecutive notes on the keyboard.
2. Do all pairs of consecutive notes have this ratio?
3. Take your answer to question 1, calculating it to three decimal places, and multiply twelve copies of this number together. Round your answer to one decimal place.
4. Explain how keyboard and piano makers solve the problem described above, using your calculations to help you.

Why do notes sound better if their frequencies are in a ratio like 3:2 or 4:3?

Pure notes are made by compressing air in a way that can be modelled by a sine wave – peaks of the wave mean the air is compressed a lot, troughs mean the air molecules are spread out. The waves $\sin x$ and $\sin 2x$ shown in the picture match up in several places. Notes an octave apart like C4 and C5 can be represented by sine waves like these: C4 is the blue curve and C5 the red curve. Notes where the sine waves don't match up well sound worse when played together.

In fact a real musical instrument uses a lot of other tricks. On a piano playing the key middle C will not only let you hear the pure tone with 261.63Hz but also – more quietly! – other tones such as C5 and G5. These are known as **harmonics**, and musical instruments sound different from each other partly because they all use harmonics in slightly different ways.

