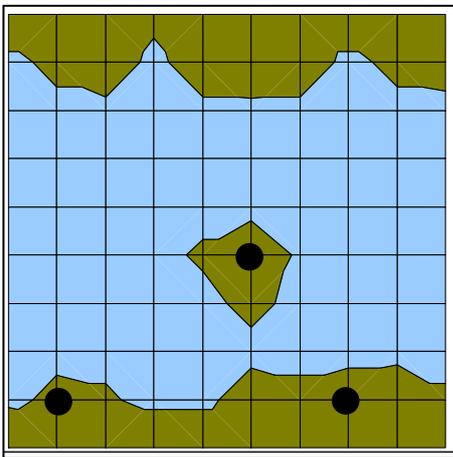


Surveying with trigonometry

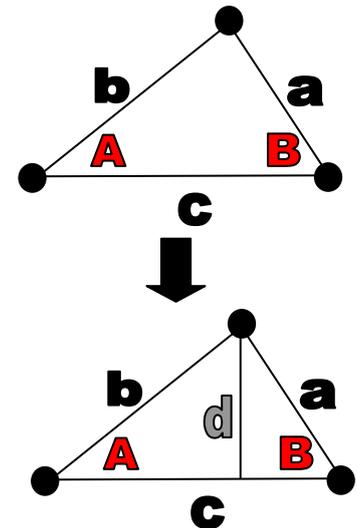
If you're an engineer trying to design a bridge to cross a river, you have to be able to say exactly where the construction company should put each of the parts for the bridge. One way of doing this is to use GPS receivers which communicate with satellites above the earth to work out exactly where on the Earth's surface the receiver is. GPS stands for Global Positioning System and is a widely used technology. But even if you have a GPS receiver, you may need to find out the position of somewhere that you can't easily get to.

Suppose that you're standing by a wide river which has an island in the middle of it.



You need to find out the distance from each of the two points marked on the river bank to the point marked on the island. But you can't get to the island easily.

Suppose that you can measure the distance c between the two points on the river bank. You can also measure two of the angles in the triangle that these three points make: A and B .



1. One way to work out the distances a and b is to split the triangle up into two smaller right angled triangles. Then we have

$$c = \frac{d}{\tan A} + \frac{d}{\tan B}$$

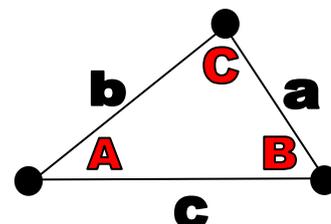
and we can rearrange this to work out an expression for d if we know A , B and c . Write down this expression.

2. Write down an expression for b in terms of A and d .
3. Write down an expression for a in terms of B and d .
4. Suppose that A is 30 degrees, B is 60 degrees and c is 5 kilometres. Calculate d and therefore a and b .
5. Suppose that A is 25 degrees, B is 75 degrees and c is 3 kilometres. Calculate d and therefore a and b .
6. Suppose that A is 45 degrees, B is 65 degrees and c is 7 kilometres. Calculate d and therefore a and b .
7. Write down some problems that might occur if you were a surveyor trying to carry out this task in a practical situation.

Surveying with trigonometry

Surveying distances

If you need to work out the distances **a** and **b** when you know the angles **A** and **B** and the distance **c**, you can split the triangle into two smaller right angled triangles provided that **A** and **B** are both acute angles (both less than 90 degrees).



A different way of calculating **a** and **b** is to use the sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

We also know that the third angle **C** of the triangle satisfies $C = 180 - (A+B)$ since angles of a triangle add up to 180 degrees.

1. To calculate **a** we need to take the first and third parts of the above equation and rearrange them to give an expression for **a** in terms of **A**, **B** and **c**. Write down this expression.
2. Write down the corresponding expression for **b** in terms of **A**, **B** and **c**.
3. Suppose that **A** is 30 degrees, **B** is 60 degrees and **c** is 5 kilometres. Calculate **a** and **b** directly.
4. Suppose that **A** is 25 degrees, **B** is 75 degrees and **c** is 3 kilometres. Calculate **a** and **b** directly.
5. Suppose that **A** is 45 degrees, **B** is 65 degrees and **c** is 7 kilometres. Calculate **a** and **b** directly.
6. The advantage of this method is that it works even when the triangle can't be split up into two smaller right angled triangles. Suppose that **A** is 105 degrees, **B** is 35 degrees and **c** is 2 kilometres. Calculate **a** and **b** directly.
7. Suppose that **A** is 125 degrees, **B** is 15 degrees and **c** is 3 kilometres. Calculate **a** and **b** directly.
8. Make a sketch of the sine curve on a graph by putting the angle **A** along the horizontal axis and $\sin A$ along the vertical axis.
9. For **A**, **B** between 0 and 180 degrees, which of the following rules is always correct?
 - A. $\sin (180 - (A+B)) = -\sin (A+B)$
 - B. $\sin (180 - (A+B)) = \sin (A+B)$
 - C. $\sin (180 - (A+B)) = 1 - \sin (A+B)$
 - D. $\sin (180 - (A+B)) = 1 / \sin (A+B)$
10. Use the correct rule to write down simplified expressions for **a** and **b**.

